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Derivation of the Tax Multiplier equation

Our starting point is to lay out the equations we use with the AE model to determine equilibrium real GDP (i.e. Y*).

C = 0.8(DI) + 4800	(C = Consumption Expenditure, DI = Disposable Income)
I = 5000	(I = Investment Expenditure)
G = 4000	(G = Government Expenditure)
X = 1000	(X = Expenditure on Exports)
M = 1000	(M = Expenditure on Imports)
T = 1000	(T = Tax Revenues)
DI = Y - T	(Y = real GDP)

If we plug these values into the equation for aggregate expenditure, then we get:

AE = [0.8(Y - 1000) + 4800] + 5000 + 4000 + (1000 – 1000)

Substituting Y for AE, we now have: Y = [0.8(Y - 1000) + 4800] + 5000 + 4000 + (1000 – 1000)

This becomes: $Y = \frac{13000}{1-0.8}$

Based on the equation above, equilibrium GDP in this case would be Y* = 65000. Figuring out equilibrium GDP is not our goal, however, our goal is to determine how equilibrium GDP changes when there is a change in taxes (T).

Let's assume that taxes are changed to become 1000 + ΔT, where ΔT is some non-specific, change in taxes. Our set of equations is now:

C = 0.8(DI) + 4800
I = 5000
G = 4000
X = 1000
M = 1000
T = 1000 + ΔT
DI = Y - T

Knowing that when solving for equilibrium here, we will ultimately substitute Y for AE within the AE equation, let's just skip straight to the step of having Y equal to the sum of expenditures.

$Y = [0.8(Y - (1000 + \Delta T)) + 4800] + 5000 + 4000 + (1000 – 1000)$

This equation reduces to: $Y = \frac{13000\text{-}(0.8 \times \Delta T)}{1\text{-}0.8}$

Our equilibrium real GDP value would be: $Y^{**} = 65000 - \frac{(0.8 \times \Delta T)}{1-0.8}$

The effect of a change in T on Y gives us ΔY, and we can determine that effect by subtracting Y* from Y** as follows:

$$\Delta Y = \left(65000 - \frac{(0.8 \times \Delta T)}{1-0.8}\right) - 65000$$

This equation can be rearranged to give us:

$$\Delta Y = \left(\frac{\text{-}0.8}{1-0.8}\right) \Delta T$$

Note that the MPC = 0.8 in this example, and if we would like to transform the specific multiplier equation above into an equation that's even more general (i.e. an equation where we could use any MPC, not just 0.8), we could rewrite this equation as:

$$\Delta Y = \left(\frac{-MPC}{1-MPC}\right) \Delta T$$